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LETTER TO THE EDITOR

Anisotropy and scaling of Eden clusters in two and three dimensions

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Abstract. For two-dimensional Eden clusters grown on flat substrates the way the perimeter per unit length and the surface width depend on the orientation of the substrate with respect to the lattice is measured. The evolution of the perimeter per unit length during growth is shown to be independent of the substrate length. For the surface width strong corrections to scaling are observed. The anisotropy of three-dimensional Eden clusters and its implications for the scaling of the surface width are investigated. The largest two- and three-dimensional clusters studied are more than one order of magnitude larger than in previous simulations.

The Eden model can be regarded as the most elementary example in the large variety of models describing the growth of random aggregates (for a review see [1]). Although by now it has been quite extensively studied it still contains aspects which are not entirely understood. Recently Freche *et al* [2] reported a weak anisotropy in the shape of two-dimensional Eden clusters grown out of a point seed on a square lattice. The diameter averaged over many independent clusters was about 1-2% larger along the lattice axes than along the diagonals. Preliminary results were also presented for Eden clusters on a simple cubic lattice. In this letter we extend their analysis in two directions.

First we describe how the surface properties of two-dimensional Eden clusters depend on the average orientation of the surface with respect to the underlying lattice. For this purpose we let Eden clusters grow on flat substrates with well defined orientation characterised by Miller indices (0, 1) (substrate parallel to a lattice axis), (1, 1) (substrate parallel to a lattice diagonal) as well as (1, 4) and (1, 2). The cluster grows in an infinitely long strip with periodic boundary conditions. This approach is complementary to the one used in [2] since it gives only indirect information about the anisotropic shape. The second part tries to improve the data of Freche *et al* [2] for Eden clusters on a simple cubic lattice. Here the clusters are grown from a point seed, and the shape anisotropy is measured.

Our data not only contain new results about the anisotropy but also new aspects concerning the scaling of the surface width which is defined as

$$w = (\langle r^2\rangle - \langle r\rangle^2)^{1/2} \qquad (1)$$

where r in our two-dimensional case is the distance of a perimeter site from the substrate and in the three-dimensional case the distance from the seed. The brackets indicate averaging over all perimeter sites of a large number of independent clusters of equal size N.

For our simulations we used version A of the Eden model in the classification introduced in [3]: all unoccupied nearest neighbours of occupied sites are called perimeter sites, and all perimeter sites have equal probability of becoming occupied in the next step of the cluster growth. In order to make optimal use of the memory capacity of the CDC Cyber 176 we used single-bit coding and in the two-dimensional case we only stored the surface region of the cluster. Thus we could study strip widths up to 20 000 and clusters with about 100 million sites. In the three-dimensional case clusters with more than 6 million sites were simulated by considering only one octant. This is similar to the use of quadrants in two dimensions [2].

The first surface property we have studied for the two-dimensional case is the number of perimeter sites N_p divided by the Euclidean length L of the substrate. There are two main results illustrated in figure 1. The perimeter per unit length, N_p/L, for fixed deposit height $h = N/L$ is practically independent of L if L varies over as much as two orders of magnitude:

$$N_p/L = f(h). \tag{2}$$

This is in marked contrast to the scaling of the surface width [3, 4]

$$w/L^{1/2} = g(h/L^z) \tag{3}$$

where the values of the exponent z mentioned before are $\frac{3}{2}$ [5], $1{\cdot}55 \pm 0.15$ [4] and 1.7 ± 0.3 [3]. The second main result is that N_p/L depends on the Miller indices. It is largest for surfaces with an average orientation parallel to a lattice axis and decreases if the surface is tilted towards the lattice diagonal. The stationary values are 2.180, 2.171, 2.153 and 2.137 with errors ± 0.007 for the Miller indices (0, 1), (1, 4), (1, 2) and (1, 1), respectively. Within the errors the ratios between these values also seem to be valid for heights smaller than the asymptotic regime.

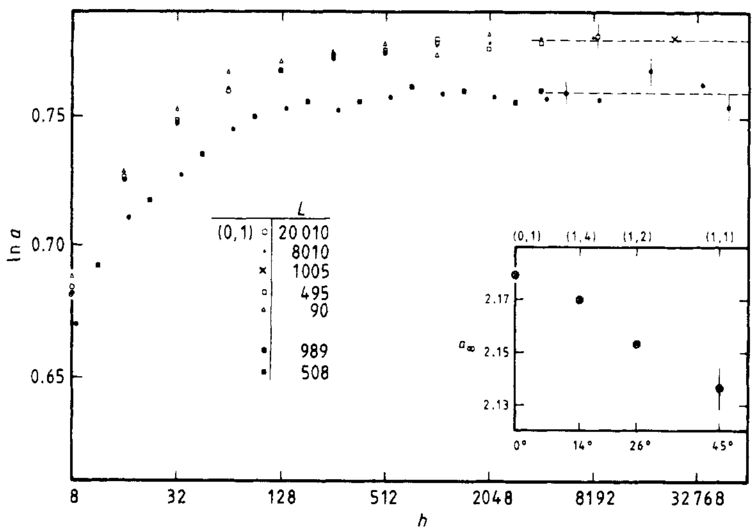

Figure 1. Number of perimeter sites per unit length $a = N_p/L$ as a function of deposit height h for the orientations (0, 1) and (1, 1) and for various substrate lengths L. Inset: asymptotic values of N_p/L for four different orientations of the substrate. The angles between the substrate normals and a lattice axis are indicated as well as the Miller indices.

In a cluster grown from a point seed, surfaces with all average orientations occur at the same time. As all perimeter sites have equal probability of becoming occupied a surface will, on average, propagate faster the higher its density of perimeter sites. It can be shown that the orientational dependence of N_p/L is thus directly related to the anisotropy of the average shape of Eden clusters via a Wulff construction [6, 7]. Our difference of 2% between N_p/L for surfaces parallel to a lattice axis or to a diagonal is thus an explanation for the shape anisotropy observed in [2].

Associated with the anisotropy of the perimeter per unit length is an anisotropy of the surface width. The larger N_p/L, the larger w is also. The ratios of the stationary values of $w/L^{1/2}$ are 0.423: 0.416: 0.405: 0.388 with errors ± 0.006 and seem to be independent of L. A more detailed account will be given in [7].

None of the values for z mentioned above was obtained for version A of the Eden model as it is hampered by particularly strong corrections to scaling compared to versions B and C [3]. As the substrate lengths we were able to study were up to a factor of 30 larger than the largest ones considered previously we tried to check the z value for version A. For this purpose we plotted the derivative $\beta(h) = d(\ln w)/d(\ln h)$ in figure 2. The data for $L = 495$ and 1005 show that for large h, when w approaches its stationary value, $\beta(h)$ goes to zero as expected. As usual [3, 4] we assume that the scaling function g in (3) grows with a power $1/2z$ for small arguments. Accordingly one expects that $\beta(h)$ goes through a plateau $1/2z$ before it drops to zero. For small values of h this plateau will be distorted by corrections to scaling. Figure 2 shows that corrections to scaling persist at least up to $h = 2000$ for intermediate and large L. One may speculate whether this can be associated with the fact that the perimeter reaches its stationary value only above this deposit height (see figure 1). The data for substrate length 8010 seem to support the z values given in the literature. One has to be careful, however. If one tentatively scales h with $L^{1.5}$ one finds that for $L = 8010$ the last value $h/L^{1.5}$ for $h = 4096$ (figure 2) is only a factor of 4 below the value $h/L^{1.5}$

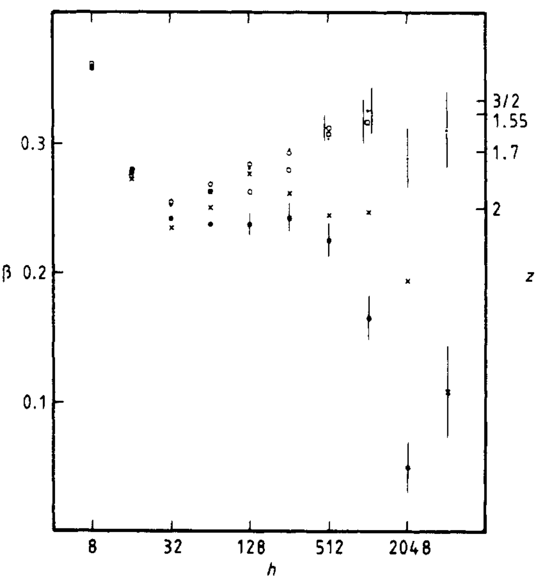

Figure 2. Exponent β for $d = 2$ against deposit height h for substrates of different lengths L and with orientation (0, 1).

which corresponds to the maximum of $\beta(h)$ for $L=495$. That means that the last data points for $L=8010$ are already quite close to the region where $\beta(h)$ starts to decrease. Therefore they should be regarded as lower bounds for the plateau value of $\beta=1/2z$ leading to an upper bound for z.

Much less is known about the scaling of the width in the three-dimensional case. It is believed [3] that for globally flat surfaces w diverges logarithmically with the substrate area. This is suggested by analogy with roughening models which works at least in two dimensions [4]. But so far there are no conclusive data nor a proof of this conjecture [3].

If one tries to investigate this question for Eden clusters grown from a point seed one has to take their anisotropy into account [2]. In order to demonstrate that three-dimensional Eden clusters are anisotropic we determined the average mean square distance $\langle r^2\rangle_j$ from the seed for perimeter sites which lie on the axes ($j=1$), on the planar diagonals ($j=2$) or on the space diagonals ($j=3$), respectively. These quantities have been normalised by the mean square distance $\langle r^2\rangle$ of all perimeter sites of the whole cluster. In figure 3 we have plotted the square roots of these ratios as functions of N. Along the axes the mean square distance of the perimeter from the seed is distinctly larger than along the planar diagonals, where it is again slightly larger than along the space diagonals. However, it is also clear that our largest clusters have not yet reached the regime of stationary growth. Therefore we have no evidence that the shape remains anisotropic for arbitrary large clusters.

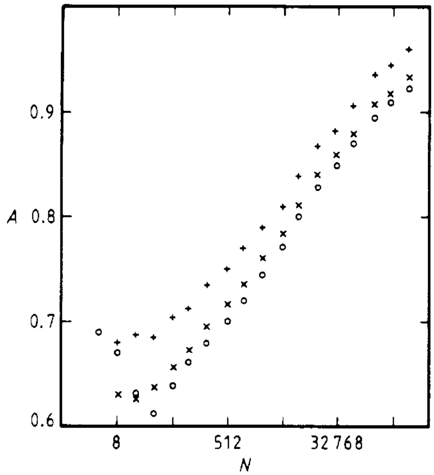

Figure 3. Anisotropy of Eden clusters for $d=3$: $A=(\langle r^2\rangle_j/\langle r^2\rangle)^{1/2}$ against cluster size N for $j=1$ (+), $j=2$ (×) and $j=3$(○).

For large anisotropic clusters it is expected that the surface width as defined in (1) is determined by the anisotropy so that w will be proportional to $N^{1/3}$. The analogous effect has been found for two-dimensional Eden clusters [2]. In order to check whether this consequence of the anisotropy can already be seen for the cluster sizes considered here we plotted the width exponent

$$\alpha(N)=\ln(w(2N)/w(N))/\ln 2 \tag{4}$$

in figure 4. It is decreasing for increasing cluster size. Six million sites are still not large enough to observe the exponent to rise again.

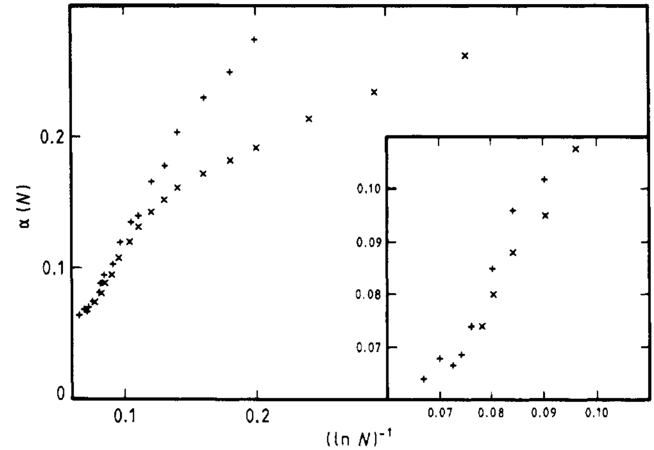

Figure 4. Exponent α for $d=3$ against the reciprocal logarithm of cluster size for simulations of whole clusters ($\times$) and octants ($+$). The inset shows the enlarged left portion of the diagram.

Figure 4 also shows that it makes a difference whether one simulates a full cluster or only one octant. The octant boundaries influence the growth very strongly. In particular the anisotropy is different in octants and in whole clusters. Hence the width is about 12% smaller than for normal growth.

Summarising our results we have given an explanation of the anisotropic shape observed for two-dimensional Eden clusters by showing that the growth velocity depends on the average orientation of the surface with respect to the underlying square lattice. As this is true for the stationary values the shape will be anisotropic for arbitrary large clusters. Three-dimensional clusters are anisotropic, too, but we could not observe whether the anisotropy reaches a stationary limit. Larger cluster sizes or a different approach are needed to settle this question. Concerning the scaling we found that the evolution of the perimeter per unit length is independent of the substrate length. It was also shown that the strong corrections to scaling of the surface width persist in model A to much larger cluster sizes than studied previously. The expected influence of the anisotropy [2, 8] on the scaling of the surface width could not be seen for our three-dimensional clusters.

After completion of this work we learned that Meakin, Botet and Jullien found similar results for two-dimensional Eden clusters grown on smaller flat substrates of different orientations.

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